A Model for Teaching Numeracy Strategies

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The New Zealand Numeracy Project is based on two kinds of number models. One kind using the Forward Number Word Sequence, the Backward Number Word Sequence, Numeral Identification, and number facts that need to automatically recalled by children are defined as knowledge models. The other using Steffe's counting types and extensions is called a strategy model. Material written to support teachers assume two different teaching models are appropriate corresponding to whether the teaching involves knowledge or strategy. This paper maps the construction of the strategy teaching model. In particular it considers the influence of Pirie-Kieran Theory and Mathematics Recovery on its development.

In New Zealand pilot numeracy projects in 2001 and 2002 (Thomas & Ward, 2001, 2002; Higgins, 2001, 2002) have led to a numeracy project for teaching children from years 1 to 8 which, as resources allow, will be available over the next few years to all New Zealand primary teachers. Teachers will be provided with assessment and teaching materials. These are extensions and modifications of the Count Me in Too materials used in 2001 (NSW Department of Education and Training, 1999a, 1999b), the work of Wright (Wright 1991a, 1991b, 1991c, 1994, 1998, Wright, Martland, & Stafford, 2000) and the work of Young-Loveridge. (Anderson, Lindsey, Shulz, Monseur & Meiers, 2002; Young-Loveridge, 1991). A major element in these projects are the forward and backward number word sequences models derived from Fuson's work (Fuson, 1988) by Wright (Wright, 1991c) and the numeral identification model created by Wright (Wright, 1991c). These are defined as *knowledge* models in the New Zealand Numeracy Project. The other kind of model was formulated from Wright's (Wright, 1991a, 1991c, 1992b) tabular overview of the development of Steffe's theory of counting types (Steffe, von Glasersfeld, Righards & Cobb; 1983; Steffe & Cobb, 1988). In this theory there is a progression of five counting types in which children solve number problems using strategies ranging from counting objects to part-whole reasoning. This is defined as a *strategy* model for the New Zealand Numeracy Project. This distinction between knowledge and strategy is similar to that used by Wright (Wright *et al.*, 2002). As the New Zealand Numeracy Project is designed to be used in the first eight years of school rather than the first three years (NSW Department of Education and Training, 1999a, 1999b) three extra stages have been added to the counting types. This set of new types represent an increasingly sophisticated sequence of part-whole stages. There has also been some slight modifications of the other counting types.

Critics who argue that splitting number into *knowledge* and *strategy* is arbitrary have a point. What may start as strategy for a child will become knowledge and to separate them is somewhat artificial. For example, suppose a child is asked the value of 8 + 8. A child says she adds 2 to 8 to make 10 and then adds 6 to give 16. She is using a part-whole strategy. But as the child improves her speed with this kind of reasoning, this method moves from using a strategy towards becoming knowledge thus blurring the distinction. However, the

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dichotomy between knowledge and strategy is maintained in the teaching materials for *pedagogical* reasons. It is assumed that teaching for knowledge and teaching for the initial development of strategy development warrant very different teaching models. This paper is concerned with the construction of a teaching model that is suitable for strategy teaching.

Assessing Number Knowledge and Strategy

All teachers involved in the New Zealand Numeracy Project are provided with enough release time to interview individually all children in their class. The interview tool is designed to determine children's number knowledge and strategies in the sense already defined.

For questions designed to determine children's ability with the forward number word sequence, backward number word sequence and numeral identification the method of recording results is typically straight forward for teachers; the questions get progressively harder in that the size of the numbers used increases and the teacher assigns a level corresponding to the highest numbers the child can reliably work with. It was a basic assumption in the writing of teacher material that teachers do find the teaching of knowledge informed by the diagnosis straight forward and well within their current models of teaching. Thus the material supplied makes no explicit reference to a model for teaching knowledge. For example, the teacher material has activities like clapping and saying every second number in order to practice the forward number sequence.

In marked contrast to assessing for knowledge teachers often find assessment of strategy using Steffe's five counting types and variations far more difficult. This because answers to questions can not used to assign a counting type or stage by simply looking at the number size. Questions can be answered correctly using a range of counting types. For example a year three child might solve 8 + 5 by a counting from one with fingers, by counting on, using a part-whole method or recalling a basic fact. The teacher must infer which counting type each child uses from a range of clues. These include whether the child uses materials, sub-vocalises when counting, speed of response, head nodding, direction of the eyes and the child's verbal explanation of the methods used. Use of recall of a known fact to solve the problem further complicates the assessment for the teacher as recall tells the teacher nothing about the child's ability to use counting types. This often represents a shift in teacher's thinking about assessment as well as an awareness raising about the different counting types children use (Hughes, 1995). Adding to the complexity a child may respond with different counting types to similar questions. For example Sharon, a year 3 student, says 8 + 7 is 15 very quickly; she explains she doubled 7 and added 1. She would be rated in New Zealand (Ministry of Education, 2002) as *Stage 5 Early Additive Part-Whole*. However, Sharon works out 9 + 6 by counting on from 9. Now Sharon could be described as *Stage 4 Advanced Counting*. In a case like Sharon's many teachers who are struggling to come to terms with assessing for strategy rather than right/wrong answers are somewhat confused by Sharon's use of two different counting types. Is she an *advanced counter* or a *part-wholer* or a mixture of both? Teachers are advised by their visiting facilitators to rate such a child at the highest counting type displayed. The reason for this advice is so the teacher can group children by the highest counting type for purposes of strategy teaching. Teachers in the project are expected to teach knowledge lessons to whole classes but strategy lessons in groups defined by the highest counting type available to the children in that group. This is in sharp contrast to teaching in England's National Numeracy Strategy. While there are a large number of similarities between the English and

New Zealand numeracy projects there are at least two core differences. In England the teacher materials do not distinguish knowledge and strategy, and materials are written for year levels with a strong emphasis on regular whole class teaching (Department for Education and Employment, 1999). In New Zealand the teacher material distinguishes between knowledge activities for whole class and strategy activities; teachers group their children by similarity of counting type and try to encourage the construction of harder and more sophisticated counting types. The merits of the English approach versus the New Zealand approach at this time is a matter for further research.

The origins and nature of the New Zealand Numeracy Project number strategy teaching model is now presented.

P-K Theory

The strategy teaching model is influenced Pirie & Kieren Theory (Pirie & Kieren, 1989, 1992, 1994; Pirie & Martin, 2000). Figure 1 shows some of the key features of Pirie & Kieren's Dynamical Theory for the Growth of Mathematical Understanding.

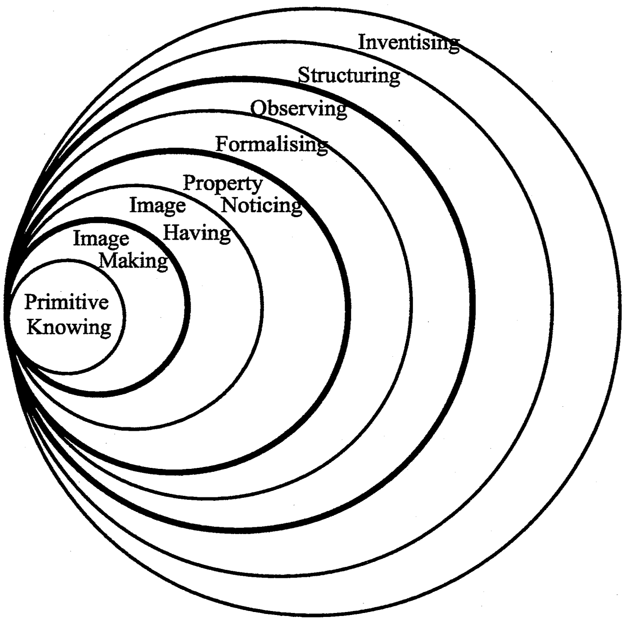

Figure 1. A dynamical theory for the growth of mathematical understanding.

Some of the stages in this model and the P-K idea of *folding back* influenced the development strategy teaching model. These influences are discussed as the various stages of the strategy teaching model is explained.

A Model for Teaching Steffe's Counting Types and their Extensions

Figure 2 shows the main features of the strategy teaching model used in the writing of teacher materials for broadening and extending children's counting types.

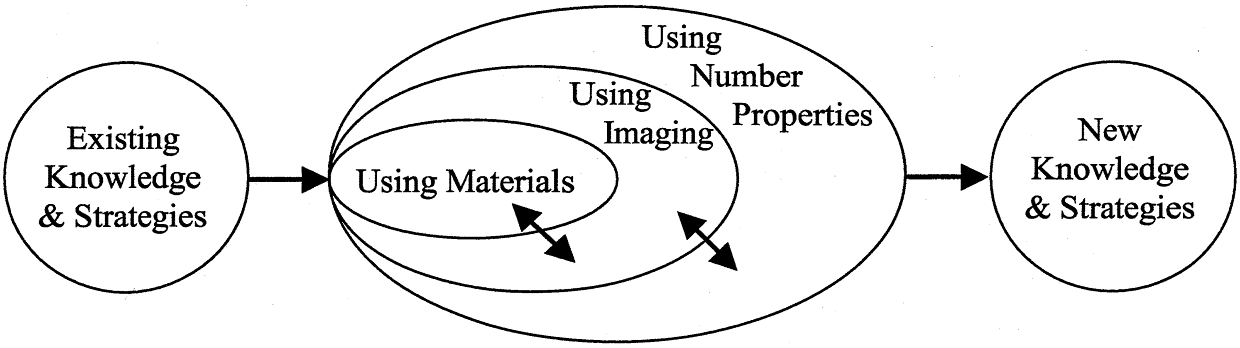

Figure 2. Strategy teaching model.

Existing Knowledge and Strategies

Existing Knowledge and Strategies is similar to *Primitive Knowing* in P-K Theory. *Primitive Knowing* is defined as "...all previously constructed knowledge, outside of the topic, that students bring to the learning of a topic" (Pirie &Martin, 2000). In the P-K theory *Primitive Knowing* is the foundation on which the *Image Making* level of understanding and later levels are built. Since *Primitive Knowing* is not a change in understanding *Existing Knowledge & Strategies* is placed outside the rings in Figure 2 to highlight its difference from these change in levels of understanding.

Using Materials

New Zealand teachers, especially those teaching children in the early years of school, have a long tradition of using materials in the teaching of mathematics. So most activities in the teacher material begin explicitly with a familiar section named *Using Materials.*

Using Imaging

The *Using Imaging* phase is a response to the real problem that many children fail to make the desired abstractions out of the *Using Materials* phase. Von Glasersfeld (1992) explains the difficulty.

> Mathematics is the result of abstraction from operations on a level on which the sensory or motor material that provided the occasion for operating is disregarded. In arithmetic this begins with the abstraction of the concept of number from acts of counting. Such abstractions cannot be *given* to students, they have to be made by the students themselves. The teacher, of course, can help by generating situations that allow or even suggest the abstraction. This is where [materials] can play an important role, but it would be naive to believe that the move from handling or perceiving objects to a mathematical abstraction is automatic. The sensory objects, no matter how ingenious they might be, merely offer an opportunity for actions from which the desired operative concepts may be abstracted; and one should never forget that the desired abstractions, no matter how trivial and obvious they might seem to the teacher, are never [obvious] to the novice. (p. 6)

Extensive use of materials without some way of promoting abstraction can lead to problems for children's learning. Ross (1989) reports for students "... even extensive experience with embodiments like base-ten blocks, and other place–value manipulatives does not appear to facilitate an understanding of place value...". In a similar vein Hart (Hart, 1989) notes the typical child who says "bricks is bricks and sums is sums" and does not connect materials with the desired abstractions. Hart notes the need for a bridge between "bricks" and "sums". The *Using Imaging* phase in the teaching model is an attempt at providing such a bridge.

Bobis (1996), arguing for the power and usefulness of visual imagery, provides a clue to a phase to follow the *Using Material* stage in the model.

> The use of concrete materials is important, but rather than moving directly from physical representations to the representations to the manipulation of abstract symbols to explain the traditional abstract procedures of algorithms, it is suggested that the emphasis be shifted to using visual imagery prior to the introduction of more formal procedures. (p. 21)

The *Using Imaging* stage where physical material is removed from the children's view is a deliberate attempt to introduce the use of visual imagery of absent objects into teaching and learning.

In P-K theory the meaning of *Image* in the *Image Making* and *Image Having* phases is more complex than the creation of picture images. Thus the *Using Imaging* phase in the teaching model is only tenuously linked to P-K Theory.

In the *Using Materials* phase the manipulatives have normally selected so that at the *Using Imaging* phase children can readily imagine the materials in their absence. For example, some activities start using counters on tens frames at the *Using Materials* phase rather than unstructured collections of counters phase because the tens frames are easier to image in their absence than an unstructured collection of counters.

Within the *Using Imaging* phase there are frequently two sub-phases. The first involves shielding or screening. This idea is derived from Mathematics Recovery (Wright, 1991a; Wright, *et al.* 2000). Shielding occurs when the teacher hides the objects that they have previously used in the *Using Material* phase so they are out of sight of the children. Then the teacher invites the children to imagine the rearrangements she is making on the material to solve the problem. For example children working in the part-whole counting type group may be asked to add 8 and 6. Children are asked to imagine what the hidden 8 counters on a tens frame look like, then to describe the groups of counters they moved around in their heads in order to get 14.

Failure to solve problems correctly at the *Using Imaging* phase may occur for a host of reasons. But whatever the reason the lack of success should make teachers aware of the fact that successful manipulation of material has *not* led to successful learning. The strategy teaching model suggests that when children do not make connections at the *Using Imaging* stage in the model that they fold back to *Using Materials*. In Figure 2 the return arrows indicating this *folding back* process. The idea of *folding back* is derived from P-K Theory (Pirie & Martin, 2000).

> When faced with a problem that is not immediately solvable at any level, an individual needs to return to an inner layer of understanding. The result of this folding back is that the individual is able to extend their current and inadequate and incomplete understanding by reflecting on and then reorganising their earlier constructs for the concept… (p. 131)

The *folding back* idea in the strategy teaching model has been altered to be an action provoked by explicit behaviour of the teacher whereas in P-K theory this is an action that i children autonomously take.

In the second sub-phase in *Using Imaging* the teachers' book provides activities in which children image without the support of shielding or folding back. Success at this stage cues the teacher to attempt to move on to the final phase of *Using Number Properties*.

Using Number Properties

In P-K theory *Property Noticing* "involves noting distinctions, combinations or connections between images, predicting how they might be achieved and recording such relationships" (Pirie & Kieren, 1992). And *Formalising* "entails consciously thinking about the noted properties, and abstracting commonalities. The person now has a class-like mental object not dependent on meaningful images" (Pirie & Kieren, 1992). In the strategy teaching model the P-K ideas in *Property Noticing* and *Formalising* led to the construction of the *Using Number Properties* phase. In this phase the objective is that children will abandon the use materials or imaging, if they indeed have not already done so, and proceed to reason directly with the numbers and their properties. The device used consistently in *Using Number Properties* phase of each learning activity is to push the number size up to the point where imaging the numbers is a burden, and solutions are best found by reasoning with abstract number properties. For example consider a group of children who have successfully used imaging and part-whole reasoning to solve problems like 9 + 16 and 25 - 7. For a problem like 7 + 89 it is difficult to *Use Imaging* but the problem is solvable simply by part-whole reasoning *when* the children are able to connect to number properties without needing images. Should children not make connections the double arrow in Figure 2 indicates *folding back* to the *Using Imaging* stage.

New Current Knowledge and Strategies

If the process has been successful children now hav*e New Current Knowledge and Strategies* have been created. The model may now be reused from the beginning.

Restrictions on the Use of the Strategy Teaching Model

In the New Zealand numeracy project Steffe's five counting types have been modified and extended to make a nine stage model; the strategy teaching stages in Table 1 shows these counting stages. In particular Steffe's last counting type *Explicitly Nested Number sequence- Part/whole Operations* which includes procedures other than counting-by-ones such as compensation, using addition to work out subtraction, and using known facts such as doubles and sums which equal ten is expanded. One stage becomes four part-whole stages to emphasise the importance of part-whole thinking and reflect the increasing sophistication in thinking required.

The shaded boxes (Table 1) indicate where *Using Imaging* or *Using Number Properties* is inappropriate because the counting type is too elementary to use these stages.

Table 1
Counting Stages and Strategy Teaching Stages

Counting Stages	Strategy Teaching Stages		
	Using Materials	Using Imaging	Using Number Properties
Emergent			
One-to-one Counting			
Counting from One on Materials			
Counting from One by Imaging			
Advanced Counting			
Early Additive Part/Whole			
Advanced Additive Part/Whole			
Advanced Multiplicative Part/Whole			
Advanced Proportional Part/Whole			

Discussion

A large scale experiment in the use of the strategy teaching model involving literally thousands of New Zealand teachers is under way this year. 75% of the teaching material supplied to teachers in the Numeracy Project in 2002 has been written to include this model. Evidence about the model's effectiveness will be gathered and reported at the end of the year. Small scale anecdotal evidence from developing the model over two years by a small group of the Numeracy Project's facilitators give cause for some optimism about the results. In particular teachers seem to readily accept the *Using Images* phase as being an obvious and natural stage that has frequently been missing in their teaching.

The teaching model is presented to teachers by their school's facilitator. For many of the facilitators the model was quite knew at the beginning of 2002 when they undertook training for facilitating the project. What remains to be seen is how these facilitators interpreted the model, and how effective they are at communicating the use of the model to their teachers.

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